Brief Revision Notes and Strategies

Straight Line	
Distance Formula	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ d is distance between A(x_1, y_1) and B(x_2, y_2)
Mid-point formula	$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ M is midpoint of A(x_1, y_1) and B(x_2, y_2)
Equation of a straight line	$y=mx+c$ $\quad \frac{y-y_1}{x-x_1}=m$ $\quad y-a=m(x-b)$
Equation of a straight line parallel to a line and through a point: e.g. parallel to $2y+3x=1$ through (4, 5)	extract gradient $\quad y=-\frac{3}{2}x+\frac{1}{2}$ $\quad m=-\frac{3}{2}$ Use $\frac{y-y_1}{x-x_1}=m$ with $m=-\frac{3}{2}$ and (4, 5)
Equation of a straight line perpendicular to a line and through a point: e.g. perpendicular to $2y+3x=1$ through (4, 5)	extract gradient: $\quad m=-\frac{3}{2}$ $\quad$ use $m_2=-\frac{1}{m_1}$ so $m=\frac{2}{3}$ use: $\frac{y-y_1}{x-x_1}=m$ with $m=\frac{2}{3}$ and (4, 5)
Equation of a perpendicular bisector	Find mid-point of line, Find gradient of line, Find perpendicular gradient Find equation – using gradient and mid-point
Equation of a median	The median of a triangle is the line from a vertex (corner) to the **mid-point** of the opposite side.
Equation of an altitude	The altitude of a triangle is the line from a vertex (corner) **perpendicular** to the opposite side.
Equation of a line parallel to x or y axis	Lines parallel to x axis are of form: y = 3 Lines parallel to y-axis are of form: x = 2
Find point of intersection of two straight lines	Solve equations of two lines simultaneously
Finding angle a line makes with x – axis	Use m = tan θ
Find angle between two lines	m = tan θ; draw triangles – sum of angles = 180°

Functions and Graphs	
Composite functions – f(g(x)) and g(f(x)) e.g. f(x) = sin x g(x) = 3x	Write: f(g(x)) = f(3x) = sin 3x and g(f(x)) = g(sin x) = 3 sin x Make sure you get them the right way round.
Simplify a composite function using algebraic fractions or rules of surds. e.g. $f(x) = 2 - x$ *and* $g(x) = \frac{2}{x}$ find $p(x)$ where $p(x) = f(g(x))$ if $q(x) = \frac{2}{2-x}$, find $p(q(x))$ in its simplest form	$p(x) = f(g(x)) = f\left(\frac{2}{x}\right) = 2 - \frac{2}{x}$ $p(q(x)) = p\left(\frac{2}{2-x}\right) = 2 - \frac{2}{\frac{2}{2-x}}$ $= 2 - 2 \times \frac{2-x}{2} = 2 - (2 - x) = 2 - 2 + x = x$
Given a function find graph of related function	y = -f(x) **reflects** graph in **x-axis** y = f(-x) **reflects** graph in **y-axis** y = f(x + a) moves graph ***a*** units to **left** y = f(x - a) moves graph ***a*** units to **right** y = f(x) + a moves graph ***a*** units **up** y = f(x) - a moves graph ***a*** units **down** More than one operation can take place e.g. y = f(x – 4) + 3 move 4 units to right and move 3 up y= 6 – f(x) reflect in x-axis and then move 6 up It is helpful to consider one transformation at a time and build up the sketch of the graph in stages.
Sketch graph of derived function	Derived function graph shows where the gradient of the function is zero. Stationary points of function are marked on x-axis of derived function graph. Look at gradient on either side of this zero to determine whether the derived function should be positive (above axis) or negative (below the axis)
Finding an inverse: eg f(x) = 5x – 1 find f^{-1} (x)	Use y = f(x) and change subject of the formula. $y = 5x - 1 \rightarrow y + 1 = 5x \rightarrow x = \frac{1}{5}(y+1)$ *Switch variables x and y; put back in function notation.* $y = \frac{1}{5}(x+1) \rightarrow f^{-1}(x) = \frac{1}{5}(x+1)$

Functions - Quadratic	
Completing the square **Ex 1.** x^2-8x+3 **Ex 2.** $2x^2+8x-1$ **Ex 3.** $3+6x-x^2$	**NB:** Coefficient of x^2 must be positive and 1 **1.** $x^2-8x+3 \rightarrow (x-4)^2-16+3 \rightarrow (x-4)^2-13$ **2.** $2x^2+8x-1 \rightarrow 2\left(x^2+4x\right)-1 \rightarrow$ $\rightarrow 2\left\{\left(x+2\right)^2-4\right\}-1 \rightarrow 2(x+2)^2-9$ **3.** $3+6x-x^2 \rightarrow -\left(x^2-6x\right)+3 \rightarrow$ $\rightarrow -\left\{\left(x-3\right)^2-9\right\}+3 \rightarrow 12-(x-3)^2$
Using for max or min and value of x for which it occurs	Since the squared term can only be positive, look for when the squared term is zero. *Using the above examples:* $(x-4)^2-13$ has a minimum of -13 at x = 4 $2(x+2)^2-9$ has a minimum of -9 at x = -2 $12-(x-3)^2$ has a maximum of 12 at x = 3
Find equation of a parabola passing through 3 points (two are roots)	If you are given two roots of a parabola i.e. where it crosses the *x*-axis - e.g. $x=a$ and $x=b$ then the equation will be given by: $y=k(x-a)(x-b)$ *k* is a constant to be determined by another point. Substitute another point into the equation to find *k*.
Discriminant Find value of k for equation to have equal roots	Equation of a quadratic is: $ax^2+bx+c=0$ $b^2-4ac=0$ equation has equal roots $b^2-4ac>0$ equation has real and distinct roots $b^2-4ac<0$ equation has no real roots. Form an equation involving *k* using the discriminant and solve to find a value for *k*.
Proofs – find range of values for *k* for which equation has real roots, no roots	Use **discriminant** to form an expression involving *k*. Investigate ranges for k for which the expression is positive or negative as required.

Functions - Polynomials	
Remainder Theorem	When $f(x)$ is divided by $(x-h)$, the remainder is $f(h)$
Factor Theorem	If $f(x)$ is a polynomial, then, $f(h) = 0 \;\Leftrightarrow\; (x-h)$ is a factor
Synthetic Division	Use for dividing a polynomial by $(x-a)$ Write down coefficients of polynomial e.g. divide: $2x^3 - x^2 - 3x - 5$ by $x - 2$ $\begin{array}{r\|rrrr} 2 & 2 & -1 & -3 & -5 \\ & \downarrow & 4 & 6 & 6 \\ \hline & 2 & 3 & 3 & \boxed{1} \end{array}$ When dividing by $(x-2)$ use 2 as the divisor When dividing by $(x+3)$ use -3 as the divisor
Given a factor find a value of k	Use synthetic division and obtain an expression for the remainder in terms of k. Put this expression equal to zero and solve for k.
Given a factor solve a cubic equation	Use synthetic division to obtain the quotient. Factorise the quadratic quotient to obtain 3 roots.
Factorise a cubic – given clues from a graph	Cubic has a factor, where the graph crosses the x-axis. Use this factor in synthetic division to find quotient. Factorise quotient to obtain 3 roots.
Solve a cubic equation with no clues	Look at constant term; use factors of it to find a factor of the polynomial. e.g. if constant is 6, try division with ±1, ±2, ±3, ±6 or find h such that f(h) = 0 using h = ±1, ±2, ±3, ±6 find quotient using synthetic division, and factorise it.
Find intersection of a line and a curve	Solve equations simultaneously. Will give rise to a quadratic or cubic
Deduce from a graph value of k for which $f(x) = k$ has 3 real roots, no real roots, etc. Deduce a solution for a quadratic inequality	Consider where the graph needs to be to cross the x-axis at 3 points, no points etc. Look at graph and determine values of x that will satisfy the inequality.

Logarithms and Exponential Functions	
Using log and exponential graphs to determine co-ordinates	Use knowledge of special logarithms: $\log_a 1 = 0$ $\log_a a = 1$ $y = a^x \Leftrightarrow \log_a y = x$
Using log and exponential graphs to determine constants in equations	Use knowledge of special logarithms: $\log_a 1 = 0$ $\log_a a = 1$ $y = a^x \Leftrightarrow \log_a y = x$ Look also for powers. e.g. $8 = 2^3$

Sequences and Recurrence Relations	
Form a recurrence relation	**Always write down the recurrence relationship** in the form $U_{n+1} = mU_n + c$ **Note:** *m* is the proportion **remaining**.
Limit formula	$L = \frac{c}{1-m}$ **providing -1 < *m* < 1** 1. **Check that the condition -1 < *m* < 1 is true** 2. **State it.**
Use the limit and interpret	The limit is the value to which the sequence will tend, in the long term. Unless you know the initial conditions, you cannot say whether the long term value is a rise or fall. State: *"In the long term,"* …*"will settle out at around"* .. If you **know** the initial value, you may deduce whether the long term value will be a decrease or an increase.
Use the limit and find the multiplier and interpret	You may be given a limit and asked to work back to find m and interpret this in the context of the question. Use the formula, find m, then interpret. *m* is what remains, so 1 – *m* is the interpretation.
Calculating terms of a recurrence relation e.g. loans, HP repayments, Bank Interest, charges	When dealing with loans and HP it is necessary to calculate each term of the sequence, to find when the loan terminates, final payment etc.
Finding limit of a sequence as n tends to infinity e.g. What is limit of sequence as n → ∞	Re-arrange into a form where the term involving n tends to zero as n → ∞ $U_n = \frac{2n+1}{n} \rightarrow \frac{2n}{n} + \frac{1}{n} \rightarrow 2 + \frac{1}{n}$ $\rightarrow \lim_{n\to\infty} 2 + \frac{1}{n} = 2 \quad as \quad \lim_{n\to\infty} \frac{1}{n} = 0$

Calculus - Differentiation	
Rules of Differentiation – algebra **Watch the signs !** **Put in straight line form**	<table><tr><th>$f(x)$</th><th>$f'(x)$</th></tr><tr><td>k *(constant)*</td><td>0</td></tr><tr><td>x</td><td>1</td></tr><tr><td>ax</td><td>a</td></tr><tr><td>x^n</td><td>nx^{n-1}</td></tr></table>
Find co-ordinates of stationary point	Differentiate the function For a SP $f'(x)=0$ Solve the equation for x co-ordinate(s) of SPs. Substitute into equation to get y co-ordinate (SV) *(often need to also determine whether max or min)* State the co-ordinates *(and nature if asked.)*
Determine nature of stationary point	1. Use table of signs. Use the **factorisation** of the **derivative** Look at signs either side of SP Maximum, minimum or point of Inflexion (PI) 2. Use 2nd derivative $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2}<0 \rightarrow$ Max: $\frac{d^2y}{dx^2}>0 \rightarrow$ Min: $\frac{d^2y}{dx^2}=0 \rightarrow$ Point of inflexion
Find the gradient of a tangent to a curve at a point P	Differentiate the equation to get gradient function. Evaluate at point P to find gradient.
Find the equation of the tangent to a curve at a given point P	Differentiate the equation to get gradient function. Evaluate at point P to find gradient. Use gradient and point to form equation.
Evaluate a derivative	Differentiate the equation to get gradient function. Evaluate at point P to find gradient. If you are working in index form – particularly with negative or fractional indices, put back into fraction and surd notation before evaluating – **CAREFULLY!**

Find co-ordinates of a point on a curve where tangent makes a specified angle with x-axis	Using m = tan θ find a value for gradient Differentiate the equation to get gradient function. Equate the value of *m* to the gradient function. Solve equation to find x co-ordinates Substitute back into equation to find y co-ordinates. State co-ordinates of the point.
Velocity & acceleration – find acceleration equation from velocity equation	Differentiate velocity to get acceleration. $a = \frac{dv}{dt}$ Evaluate at given value of t for acceleration.
Distance & velocity – find velocity equation from distance equation	Differentiate distance to get velocity. $v = \frac{ds}{dt} \quad or \quad v = \frac{dx}{dt} \quad or \quad v = \frac{dh}{dt}$ Evaluate at given value of t for velocity.
Using height and velocity	Greatest height is when velocity = 0
Optimisation problem Find value of x that makes function max or min e.g. Find max or min surface area, volume, profit etc.	This is really a stationary point question 1. Find a constraint (if required) 2. Differentiate the function 3. For a SP $f'(x) = 0$ 4. Solve the equation for to find x **5. Show that it is a max or min as required.** 6. Find other variables in question if required. 7. Interpret the solution

Calculus - Integration

Rules of Integration – algebra

Watch the signs !

+ constant of integration *c* *!!!!!*

Put in straight line form

$f(x)$	$\int f(x)\,dx$
k (constant)	kx
x^n	$\dfrac{x^{n+1}}{n+1}$

Solve a differential equation given a point that the equation passes through.

Integrate – include the constant of integration.
Substitute the given point in the equation
Find the constant c of integration.
Write out the solution including the constant found.

Ex. 1 Find y(x) if $\dfrac{dy}{dx} = x^3 + x^2 - 1$
and y(x) passes through (1, 1)

$$\frac{dy}{dx} = x^3 + x^2 - 1 \quad \rightarrow \quad y = \frac{x^4}{4} + \frac{x^3}{3} - x + c$$

$$when\ \ x = 1,\ y = 1 \quad \rightarrow \quad 1 = \tfrac{1}{4} + \tfrac{1}{3} - 1 + c$$

$$1\tfrac{5}{12} = c \qquad \Rightarrow \qquad y = \frac{x^4}{4} + \frac{x^3}{3} - x + 1\tfrac{5}{12}$$

Integrate a function

Algebraic function requiring putting into straight line form.

Prepare to integrate – put in straight line form

$$\int \frac{(x^2-4)(x^2+4)}{x^3}\,dx \quad \rightarrow \int \frac{(x^4-16)}{x^3}\,dx$$

$$\rightarrow \int \frac{x4}{x^3} - \frac{16}{x^3}\,dx \quad \rightarrow \quad \int x - 16x^{-3}\ \ dx$$

Evaluate a definite integral

Integrate – no need for constant of integration
Evaluate the expression at upper limit
Subtract value of expression at lower limit.

$$\int_1^3 2x^2\,dx \quad \rightarrow \quad \left[\frac{2x^3}{3}\right]_1^3 \quad \rightarrow \quad \left(\frac{2\times3^3}{3}\right) - \left(\frac{2\times1^3}{3}\right)$$

$$\rightarrow \quad 18 - \tfrac{2}{3} = 17\tfrac{1}{3}$$

Calculate area under a curve

Find limits of integration if not given
Integrate as a definite integral.

Area under the x-axis is negative and must be treated separately.

Calculate area between two curves or between a straight line and a curve	Find limits of integration if not given. Use simultaneous equations. Area = ∫ upper curve - ∫ lower curve dx Simplify the expression before integrating Integrate as a definite integral. Evaluate for area. ***Do not need to make allowance for area below x-axis, it takes care of itself automatically.***
Using a derivative to assist in integrating an unfamiliar function	Differentiate a similar function (given) Look at any difference from function to be integrated. Deduce original function by use of constants.
Velocity & acceleration – find velocity equation from acceleration equation	Integrate acceleration to get velocity. $a = \frac{dv}{dt} \quad so \quad v = \int a \ dt \quad \to \quad v = \int \frac{dv}{dt} \ dt$ Evaluate at given value of t for velocity or to find constant of integration.
Distance & velocity – find distance equation from velocity equation	Integrate velocity to get distance (s). $v = \frac{ds}{dt} \quad so \quad s = \int v \ dt \quad \to \quad s = \int \frac{ds}{dt} \ dt$ Evaluate at given value of t for distance or to find constant of integration.

<table>
<tr><th colspan="2">Trigonometry</th></tr>
<tr><td>Writing down equation of a trig function using amplitude, periodicity and phase angle</td><td>amplitude: max – height of wave from zero
periodicity: how many waves in 360° or 2π
phase angle: Is wave shifted left or right
e.g. $\boldsymbol{a \sin nx}$
a = amplitude n = waves in 360°
e.g. $\mathbf{3\cos(x-30)}°$
amplitude = 3 phase angle = 30° (moved right 30°)</td></tr>
<tr><td>Interpreting a solution to a trig graph</td><td>Look at the graph
– identify your solution as a point on the graph.
Value of x in degrees or radians is on x-axis.</td></tr>
<tr><td>Compound Angle formulae

Double Angle formula:
(put A = B)</td><td>$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$= 1 - 2\sin^2 A$
$= 2\cos^2 A - 1$</td></tr>
<tr><td>Other Trigonometric formula</td><td>$\sin^2 A + \cos^2 A = 1$
$\dfrac{\sin A}{\cos A} = \tan A$</td></tr>
<tr><td>Exact Values</td><td>
<table>
<tr><td></td><td>30°</td><td>45°</td><td>60°</td></tr>
<tr><td></td><td>π/6</td><td>π/4</td><td>π/3</td></tr>
<tr><td>sin</td><td>$\frac{1}{2}$</td><td>$\frac{1}{\sqrt{2}}$</td><td>$\frac{\sqrt{3}}{2}$</td></tr>
<tr><td>cos</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{1}{\sqrt{2}}$</td><td>$\frac{1}{2}$</td></tr>
<tr><td>tan</td><td>$\frac{1}{\sqrt{3}}$</td><td>1</td><td>$\sqrt{3}$</td></tr>
</table>
</td></tr>
</table>

Using exact values Compound angles, Double angles, Right angle triangles Find sin(p + q) etc	Use Pythagoras to determine hypotenuse Write down sin p = …. cos p = …. etc, Write down the appropriate formula **USE THE FORMULA SHEET !** Substitute values and simplify.
Solve a trig equation Using common factor	**Ex.** **sin 2x – sin x = 0** *Replace sin 2x with 2 sin x cos x* 2 sin x cos x – sin x = 0 sin x (2cos x – 1) = 0 No constant term → it is always common factor.
Solve a trig equation Factorising a quadratic in cos x or sin x	**Ex.** **cos 2x – sin x = 0** *Replace cos 2x with* $1 - 2\sin^2 x$ $1 - 2\sin^2 x - \sin x = 0$ $2\sin^2 x + \sin x - 1 = 0$ (2 sin x – 1)(sin x + 1) = 0 Constant term → it is always two brackets

When replacing with $2\cos^2 x - 1$ or $1 - 2\sin^2 x$ use brackets.

NEVER divide by cos x or sin x in a quadratic or common factor equation.

Solving Trigonometric Equations	
Equations with **sin 2x** and **sin x** or **sin 2x** and **cos x**	Replace sin 2x with 2 sin x cos x Then factorise – common factor
Equations with **cos 2x** and **sin x**	Replace cos 2x with $1 - 2\sin^2 x$
Equations with **cos 2x** and **cos x**	Replace cos 2x with $2\cos^2 x - 1$
Equations with $\mathbf{\cos^2 x}$ and **cos x** or $\mathbf{\sin^2 x}$ and **sin x**	If there is a constant term – factorise (….)(……) If no constant term, then common factor
Equations with $\mathbf{\cos^2 x}$ and **sin x** or $\mathbf{\sin^2 x}$ and **cos x**	Replace $\cos^2 x$ with $1 - \sin^2 x$ Replace $\sin^2 x$ with $1 - \cos^2 x$
Equations with $\mathbf{\cos^2 x}$ only or $\mathbf{\sin^2 x}$ only	Re-arrange → $\cos^2 x = \ldots$ take square root. Re-arrange → $\sin^2 x = \ldots$ take square root Remember $\pm\sqrt{}$ there will be 4 solutions
Equations with **cos 2x** only or **sin 2x** only	Re-arrange → cos 2x = … Find 2 values for 2x Re-arrange → sin 2x = … Find 2 values for 2x Do not forget extended domain. $0 \le x \le 360°$ becomes $0 \le 2x \le 720°$
Equations of type **R cos (x – a)**	Solve to find two angles for (x – a) Then solve these to find values for x.
Equations with **a cos x + b sin x = 0**	Divide by cos x to get: $\tan x = -\frac{a}{b}$ Solve as a basic equation